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ALGORITHMS FOR CONSTRUCTING A SET-VALUED MAPPING IN R^3

In many applied problems, there is a need to construct convex sets, but if these sets are obtained as a solution to a mathematical model, then the concept of a support function is most often used. Although this function is defined only for convex sets, it can also be constructed for convex hulls of compact sets. With the help of this function, it is possible to introduce the concepts of differentiation and integration in the space of convex compact

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sets. The problem arises of constructing a convex hull of a set if the values of its support function in some directions are known. Three algorithms can be used for this. The first uses the property of the support function and constructs the set through the intersections of hyperplanes defined by the support function. The second algorithm uses the values of the Minkowski functional, which is related to the support function. The third constructs the set according to the meaning of the deformation function, which is also related to the values of the support function. In this article, the task is to compare the construction speed for these three algorithms. The Intel Core i5-13450HX processor was used for calculations. Also in this article, a numerical method for constructing a solution for equations with a set-valued right-hand side, similar to the Euler method, is implemented.

MSC: 34A60, 34A99, 68W25.

Keywords: support function, deformation function, Minkowski functional, convex sets, initial problem with a multivalued right-hand side.

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Introduction

Differential equations with a multivalued right-hand side are widely used as mathematical models of processes and phenomena of various nature.

The solution of such an equation is a multivalued mapping in the space $compR^n$. Studies of the existence of solutions and their properties can be found, for example, in works [1–10]. At the same time, the issue of constructing solutions by numerical methods is actual. Since the solution of a differential equation with a multivalued right-hand side is a convex set, first of all, algorithms for constructing a set in space are needed. This can be done using the support function of the set [11–13], the deformation function [14; 15] or the Minkowski functional [15–19]. Secondly, it is necessary to construct an algorithm for constructing a sequence of sets that are solutions of the considered differential equation with a multivalued right-hand side at successive moments of time. An analogue of the Euler method is one of the most widely used numerical methods applied to this type of equations. In [20] was presented an estimate for the Hausdorff distance between the set of solutions of a differential inclusion and the set of solutions of its Euler discrete approximation, using an averaged modulus of continuity for multifunctions.

This article presents an algorithm for constructing solutions to differential equations with the Hukuhara derivative: in three-dimensional space by numerical methods that are based on the Euler method, but differ in the methods of constructing the boundary points of the convex set - using support functions, deformation functions, and the Minkowski functional.

Main Results

1. Construction of a convex hull using the intersection of support hyperplanes

Let's construct the boundary of a convex set in three-dimensional space $F \in comp(\mathrm{R}^3)$. To construct it, we will use the value of the support function for the three-dimensional set.

The support function is determined

$$c(f,\psi) = \underset{f\in F}{max}(f,\psi). \tag{1}$$

The maximum on the right-hand side is achieved because the scalar product (f,ψ) is continuous in f and the set F is compact.

1.1. Algorithm for constructing the convex hull of a set using support function

Let's introduce the support vector

$$\psi = \begin{pmatrix} \sin\theta * \cos\varphi \\ \sin\theta * \sin\varphi \\ \cos\theta \end{pmatrix} \tag{2}$$

is the set of all vectors of unit length expressed in spherical coordinates relative to two angles θ, φ, where $\theta \in [0,\pi]$ $\varphi \in [0, 2\pi]$. Let us take $N/2$ uniformly distributed points from first interval $\theta_i \in [0,\pi], i = \overline{0, N/2}$ and N points from second one, i.e. $\varphi_j \in [0, 2\pi], j = \overline{0, N}$.

There are vectors ψ on the sphere are arranged in rows, and the poles are represented by vectors ψ with $\theta = 0,\ \theta = \pi$. For first row we can construct the matrices with the values of vectors:

$$M = (\psi_1, \psi_2, \psi_3)^T$$

There is $\psi_1 = \psi|_{\theta=0}$ and $\psi_{2,3} = \psi|_{\theta=\frac{2\pi}{N}, \varphi=\overline{0,2\pi}}$.

Further, for each angles $\theta = \overline{0 + \frac{2\pi}{N}, \pi - \frac{2\pi}{N}}$ we construct two matrices M_1, M_2. These matrices contain rows of points on a two-dimensional sphere that are the vertices of triangles from sphere triangulation.

$$M1 = (\psi_1, \psi_2, \psi_3)^T$$

$$\psi_1 = \begin{pmatrix} \sin\theta_i * \cos\varphi_j \\ \sin\theta_i * \sin\varphi_j \\ \cos\theta_i \end{pmatrix}, \ \psi_2 = \begin{pmatrix} \sin\theta_{i+1} * \cos\varphi_j \\ \sin\theta_{i+1} * \sin\varphi_j \\ \cos\theta_{i+1} \end{pmatrix}, \ \psi_3 = \begin{pmatrix} \sin\theta_{i+1} * \cos\varphi_{i+1} \\ \sin\theta_{i+1} * \sin\varphi_{i+1} \\ \cos\theta_{i+1} \end{pmatrix},$$

$$M2 = (\psi_1, \overline{\psi_2}, \psi_3)^T \text{ where } \overline{\psi_2} = \begin{pmatrix} \sin\theta_i * \cos\varphi_{j+1} \\ \sin\theta_i * \sin\varphi_{j+1} \\ \cos\theta_i \end{pmatrix}$$

where $\theta_i \in \left[0 + \frac{2\pi}{N}, \pi - \frac{2\pi}{N}\right], \varphi_j \in [0, 2\pi]$.

Respectively, for the lower pole we have

$$M = (\psi_1, \psi_2, \psi_3)^T$$

There is $\psi_1 = \psi|_{\theta=\pi}$ and $\psi_{2,3} = \psi|_{\theta=\pi-\frac{2\pi}{N}, \varphi=\overline{0,2\pi}}$

For all matrices for each row, we calculate the value of the support function and we solve the systems of linear equations

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}^T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} c(F,\psi_1) \\ c(F,\psi_2) \\ c(F,\psi_3) \end{pmatrix}.$$

After solving the system of linear equation, we obtain a vector x that lies on the boundary of the convex hull of the set F. These points are the intersection points of hyperplanes with normal vectors ψ_1, ψ_2, ψ_3.

1.2. Numerical calculations for support function

Construction of the convex hull of a set, using the support function. For which of the sets we use N=60. That means we have the 1800 support vectors for each set.

The sphere $F = S_3(5, -5, 5)$ are built

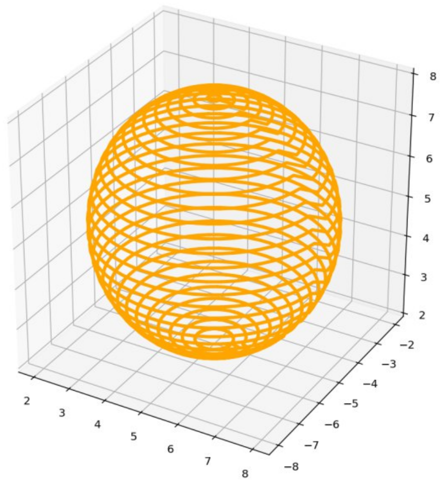

Рис. 1: $c(F,\psi) = 3\|\psi\| + 5\psi_1 - 5\psi_2 + 5\psi_3$.

The time it took to calculate all the points to construct the convex hull of the set is 0.2328968048095703.

The cube $F = K_{3,3,3}(5,-5,5)$ are built

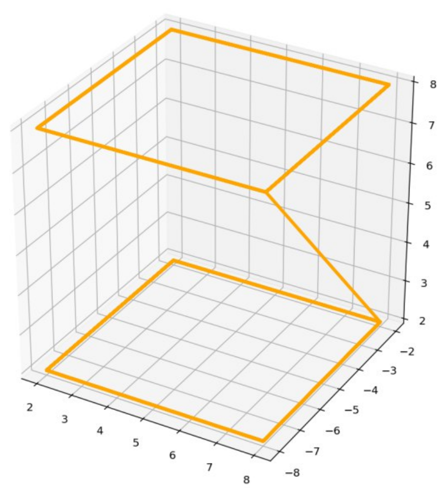

Рис. 2: $c(F,\psi) = 3(|\psi_1| + |\psi_2| + |\psi_3|) + 5\psi_1 - 5\psi_2 + 5\psi_3$

The time it took to calculate all the points to construct the convex hull of the set is 0.2111680507659912.

Let us construct the convex hull of the sum of two sets $S_3(5,-5,5)$ and $K_{3,3,3}(0,0,0)$ using the intersection of the supporting hyperplanes

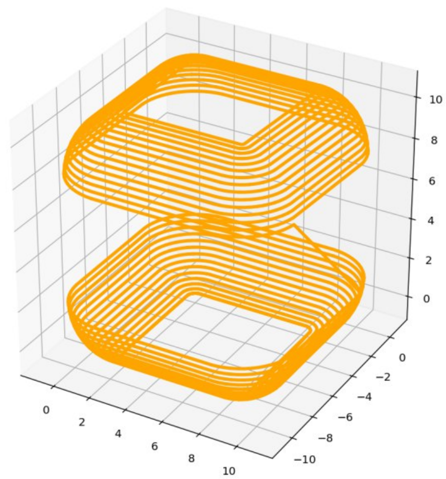

Рис. 3: $c(F,\psi) = 3\|\psi\| + 3(|\psi_1| + |\psi_2| + |\psi_3|) + 5\psi_1 - 5\psi_2 + 5\psi_3$

The algorithm time working is 0.2328968048095703.

2. Construction of a convex hull using the deformation function

The deformation function of a convex set $F \in conv(\mathbf{R}^n), 0 \in intF$ is called the function

$$d(F,\psi) = sup\{\lambda > 0 : \lambda\psi \in A\}, \psi \in S$$

The deformation function coincides with the inverse Minkowski functional on the unit sphere.

Using the deformation function, the set $F \in conv(\mathbf{R}^n)$ can be represented in the form

$$A = \bigcup_{\varphi \in S} \{x \in \mathbf{R}^n : x = \lambda\phi, \lambda \in [0, d(F,\varphi)]\}\,; \text{ S—the unit sphere} \qquad (3)$$

2.1. Algorithm for constructing the convex hull of a set using the deformation function

1. Generate a set of directions ψ uniformly distributed on the unit sphere. We set the values of the angles θ, φ, where $\theta \in [0, \pi]$ $\varphi \in [0, 2\pi]$ to determine the support vectors (2) in the spherical coordinate system.

2. Next we define the area of space in which the given set lies. This area is a rectangle for which we find the center. To do this, we find the values of the support function $c(F, \psi)$ from support vectors (2) with $\theta = 0$, $\theta = \pi$ and $(\theta = \frac{\pi}{2}; \varphi = 0)$, $(\theta = \frac{\pi}{2}; \varphi = \pi)$ $(\theta = \frac{\pi}{2}; \varphi = \frac{\pi}{2})$ $(\theta = \frac{\pi}{2}; \varphi = \frac{3\pi}{2})$, then we obtain the center point with the arithmetic mean of the values of the support function from opposite support vectors.

3. We use the algorithm for finding the deformation function. It calculates vector length in each given direction, so that it still remains inside the figure.

4. Next, for each support vector ψ we calculate values of support function minus support function from center point.

5. Fix the support vector $\psi_i \in \psi$ and for each other support vector ψ if the $(\psi_i, \psi) > 0$ calculate $\lambda = \underset{\psi}{min} \left\{ \frac{c(F,\psi)}{(\varphi,\psi)} \right\}$. This lambda guarantees us that the vector will satisfy all conditions and will not go beyond the boundary of the convex hull in any direction.

6. We construct a vector $\lambda \cdot \varphi + c$ that lies on the boundary of the convex hull, where c is a center vertor.

2.2. Numerical calculations for deformation function

Construction of the convex hull of a set, using the deformation function. For which of the sets we use N=60. That means we have the 1800 support vectors for each set.

The sphere $F = S_3(5, -5, 5)$ are built

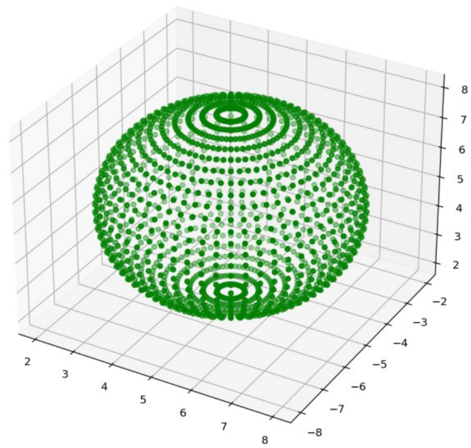

Рис. 4: $c(F, \psi) = 3\|\psi\| + 5\psi_1 - 5\psi_2 + 5\psi_3$.

The time it took to calculate all the points to construct the convex hull of the set is 9.219122409820557.

The cube $F = K_{3,3,3}(5, -5, 5)$ are built

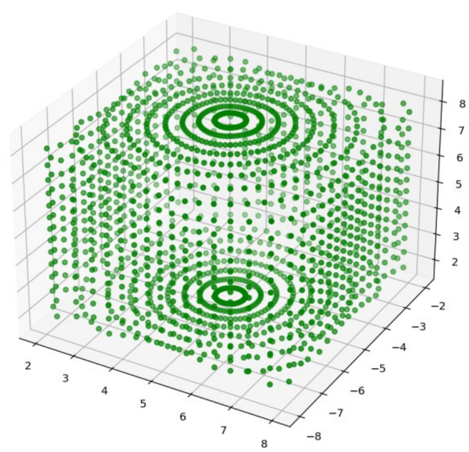

Рис. 5: $c(F, \psi) = 3(|\psi_1| + |\psi_2| + |\psi_3|) + 5\psi_1 - 5\psi_2 + 5\psi_3$

The time it took to calculate all the points to construct the convex hull of the set is 9.053423881530762.

Let us construct the convex hull of the sum of two sets $S_3(5, -5, 5)$ and $K_{3,3,3}(0, 0, 0)$ using the deformation function

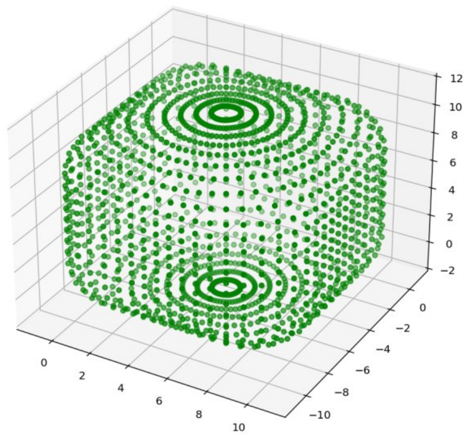

Рис. 6: $c(F, \psi) = 3\|\psi\| + 3(|\psi_1| + |\psi_2| + |\psi_3|) + 5\psi_1 - 5\psi_2 + 5\psi_3$

The algorithm time working is 9.10199236869812.

Also, we construct the convex hull of the tetrahedron

$$co\left\{\begin{pmatrix}1\\1\\0\end{pmatrix};\begin{pmatrix}5\\1\\0\end{pmatrix};\begin{pmatrix}2.5\\5\\0\end{pmatrix};\begin{pmatrix}2.5\\2.5\\5\end{pmatrix}\right\}$$

using the deformation function

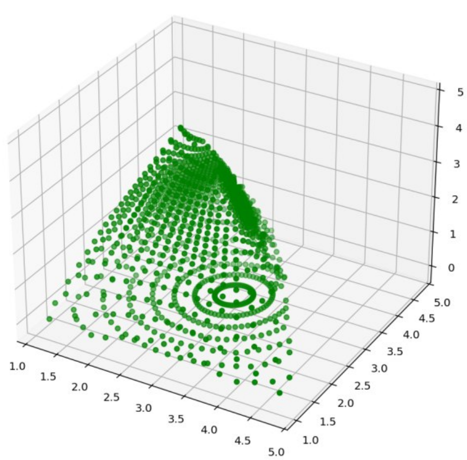

Рис. 7: $c(F, \psi) = max\,\{\psi_1 + \psi_2; 5\psi_1 + \psi_2; 2.5\psi_1 + 5\psi_2; 2.5\psi_1 + 2.5\psi_2 + 5\psi_3\}$

The algorithm time working in this case is 9.07657241821289.

3. Minkowski functional

The Minkowski functional of a convex set $F \in conv(\mathbf{R}^n), 0 \in intF$ is the function

$$m(x, F) = inf\{\lambda > 0 : \frac{x}{\lambda} \in F\}. \quad (4)$$

Using the Minkowski functional, the set $F \in conv(\mathbf{R}^n)$ can be represented in the form

$$F = \{x \in \mathbf{R}^n : m(x, F) \leq 1\}. \quad (5)$$

It follows that it is necessary to check the inequality $(x, \psi) \leq c(F, \psi)$, for a certain set of points.

3.1. Algorithm for constructing the convex hull of a set using the Minkowski functional

1. We set the values of the angles θ, φ, where $\theta \in [0, \pi]$ $\varphi \in [0, 2\pi]$ to determine the support vectors (2) in the spherical coordinate system.
2. We define the support function $c(F, \psi)$ of the set $F \in \mathbf{R}^n$ we are constructing using a formula.
3. Next we define the area of space in which the given set lies. This area is a rectangle for which we find the center. To do this, we find the values of the support function in the directions (2) with $\theta = 0$, $\theta = \pi$ and $(\theta = \frac{\pi}{2}; \varphi = 0)$, $(\theta = \frac{\pi}{2}; \varphi = \pi)$ $(\theta = \frac{\pi}{2}; \varphi = \frac{\pi}{2})$ $(\theta = \frac{\pi}{2}; \varphi = \frac{3\pi}{2})$, then add 1 to each of the values found in the corresponding directions.
4. Next, we construct a grid of points lying between the planes defined by these constraints; this grid is specified using the step parameter. This parameter specifies the splitting step.
5. Next, for each direction ψ, we subtract the value of the support function of the center in this direction of the rectangle found earlier from the value of the support function of the set in this direction.
6. For each point x, we check for which support vecotrs ψ the inequality$(x, \psi) \leq c(F, \psi)$, if it is satisfied, and points with a value of the Minkowski functional less than or equal to one are saved for further construction.
7. We construct the resulting points.

3.2. Numerical calculations for the Minkowski functional

Construction of the convex hull of a set, using the Minkowski functional. For which of the sets we use N=60. That means we have the 1800 support vectors for each set.

The sphere $F = S_3(5, -5, 5)$ are built using the step =0.24, nearly 18000 points

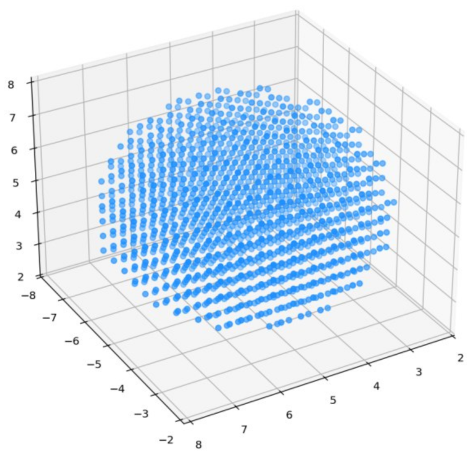

Рис. 4: $c(F, \psi) = 3\|\psi\| + 5\psi_1 - 5\psi_2 + 5\psi_3$.

The time it took to calculate all the points to construct the convex hull of the set is 7.376594305038452.

The cube $F = K_{3,3,3}(5, -5, 5)$ are built using the step =0.24, nearly 18000 points

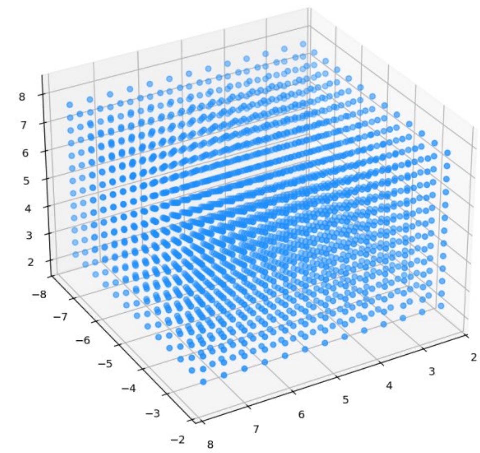

Рис. 5: $c(F,\psi) = 3(|\psi_1| + |\psi_2| + |\psi_3|) + 5\psi_1 - 5\psi_2 + 5\psi_3$

The time it took to calculate all the points to construct the convex hull of the set is 10.607192277908325.

Let us construct the convex hull of the sum of two sets $S_3(5,-5,5)$ and $K_{3,3,3}(0,0,0)$ using the step =0.7, nearly 18000 points

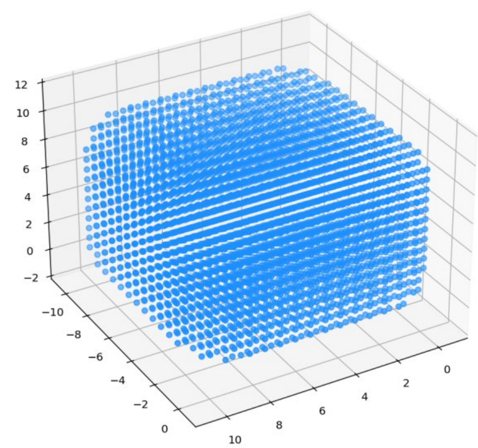

Рис. 6: $c(F,\psi) = 3\|\psi\| + 3(|\psi_1| + |\psi_2| + |\psi_3|) + 5\psi_1 - 5\psi_2 + 5\psi_3$

The algorithm time working is 12.864967107772827.

Also, we construct the convex hull of the tetrahedron

$$co\left\{ \begin{pmatrix}1\\1\\0\end{pmatrix}; \begin{pmatrix}5\\1\\0\end{pmatrix}; \begin{pmatrix}2.5\\5\\0\end{pmatrix}; \begin{pmatrix}2.5\\2.5\\5\end{pmatrix} \right\}$$

using the Minkowski functional with step =0.24, nearly 18000 points

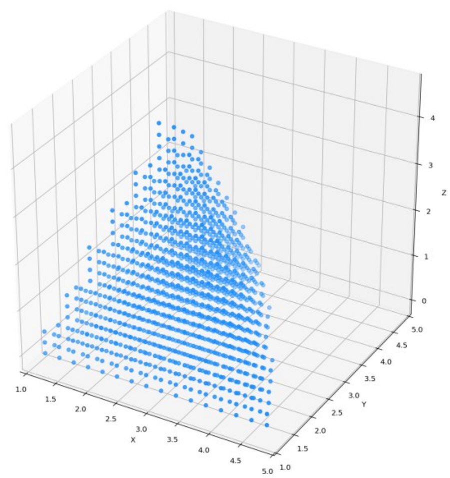

Рис. 7: $c(F, \psi) = max\,\{\psi_1 + \psi_2; 5\psi_1 + \psi_2; 2.5\psi_1 + 5\psi_2; 2.5\psi_1 + 2.5\psi_2 + 5\psi_3\}$

The algorithm time working in this case is 12.810363054275513.

But we can't see the upper point in this graphics, and we calculate some special case. It is 2016000 points (step=0.05) and for each we use the 60 support vectors

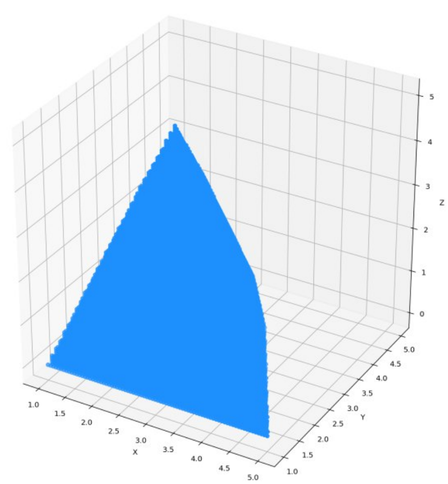

Рис. 8: $c(F, \psi) = max\,\{\psi_1 + \psi_2; 5\psi_1 + \psi_2; 2.5\psi_1 + 5\psi_2; 2.5\psi_1 + 2.5\psi_2 + 5\psi_3\}$

The algorithm time working in this case is 1249.7926468849182.

4. Constructing of the solve of the Hukuhara initial problem

We define Hukuhara derivative as the setvalued mapping $X : \mathbf{R}^1 \to comp(\mathbf{R}^n)$ which we call the Hukuhara derivative at the point $t_0 \in \mathbf{R}^1$ if there exist differences $X(t_0+\Delta t)\overset{h}{-}X(t_0)$ and $X(t_0)\overset{h}{-}X(t_0-\Delta t)$ for all small $\Delta t > 0$ and there exists an element $D_hX(t_0) \in conv(\mathbf{R}^n)$ such that

$$\lim_{\Delta t\to 0+} h\left(\frac{X(t_0+\Delta t)\overset{h}{-}X(t_0)}{\Delta t}, D_hX(t_0)\right) =$$

$$= \lim_{\Delta t\to 0+} h\left(\frac{X(t_0)\overset{h}{-}X(t_0-\Delta t)}{\Delta t}, D_hX(t_0)\right)$$

Let's introduce the initial problem with a set-valued the right-hand side

$$\begin{aligned} &D_h\chi = A(t)\chi(t) + F(t),\\ &\chi(0) \equiv \chi_0. \end{aligned} \tag{6}$$

In the problem (6) the last term we take as: $A(t) \in \mathrm{R}^{n\times n}$, F(t) — setvalued convex mapping, $t \in [0,T] \subset \mathrm{R}$.

We divide the interval into k subintervals and represent the solve in integral form as

$$\chi_{k+1}(t) = \chi_0 + \int_0^T [A(t)\chi(t) + F(t)]\, ds,\ k = 0, 1, ...$$

$$\chi_0(t) \equiv \chi_0.$$

Using the properties of the support function and applying Euler's method, we obtain

$$c(\chi_m(t_{k+1}), \psi_i) = c(\chi_m(t_k), \psi_i) + \delta\cdot\left[c\left(\chi_m(t_k), A^T(t_k)\,\psi_i\right) + c\left(F(t_k), \psi_i\right)\right],$$

where we define the second term as

$$c\left(\chi_m(t_k), A^T(t_k)\,\psi_i\right) = \begin{cases} 0 & \text{if } A^T(t_k)\,\psi_i = 0,\\ \left\|A^T(t_k)\,\psi_i\right\| c\left(\chi_m(t_k), \frac{A^T(t_k)\psi_i}{\|A^T(t_k)\psi_i\|}\right) & \text{if } A^T(t_k)\,\psi_i \neq 0. \end{cases}$$

Also, that $A^T(t_k)\,\psi_i \neq 0$ we obtain

$$c(\chi_m(t_{k+1}), \psi_i) \approx c(\chi_m(t_k), \psi_i) + \delta\left[\left\|A^T(t_k)\,\psi_i\right\| c(\chi_m(t_k), \tilde{\psi_{ik}}) + c\left(F(t_k), \psi_i\right)\right].$$

where $\tilde{\psi_{ik}}$ we find from the condition

$$\left\| \tilde{\psi_{ik}} - \frac{A^T(t_k)\,\psi_i}{\|A^T(t_k)\,\psi_i\|} \right\| = \min_{j=\overline{0,p-1}} \left\| \psi_j - \frac{A^T(t_k)\,\psi_i}{\|A^T(t_k)\,\psi_i\|} \right\|.$$

Let solve the example of this type of initial problem

$$D_h X = A(t)X + F(t)$$

with $A(t) = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$; $X_0 = 5\|\psi\|$ is the initial set and $F(t) = t\|\psi\|$ is a set-valued mapping. In algorithm we use the value of parameters t=0.2, delta =0.2. In first case we use the 1800 support vectors for building a set in every step.

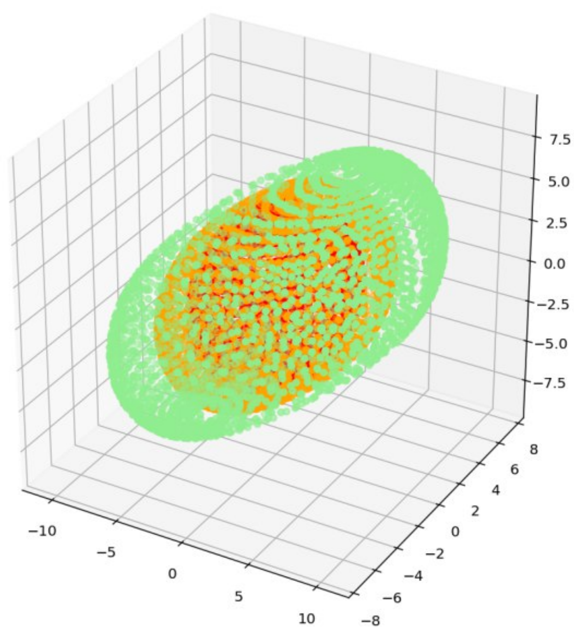

Рис. 9. The solution of initial problem using methods of intersection of support hyperplanes.

The algorithm time working in this case is 721.9192731380463.

Also, was built the case with 595 support vectors.

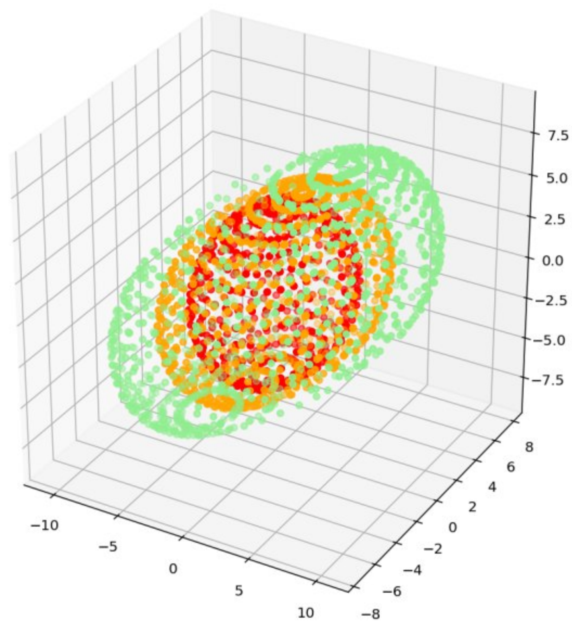

Рис. 10. The solution of initial problem using methods of intersection of support hyperplanes.

Conclusion

Thus, sets were constructed using three methods. The first used sequential hyperplane intersections, the second used the values of the Minkowski functional, and the third used the deformation function. Also, in the case of hyperplane intersections and the deformation function, solutions to the initial problems with a set-valued right-hand side and Hukuhara derivative were constructed using the Euler method and the scheme presented in the article by Plotnikov and Skripnik, only generalising it for the three-dimensional case.

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Бондаренко К. С., Таірова М. С., Кічмаренко О. Д., Вербіцький В. В.
Алгорими побудови множиннозначного відображення в R^3

Резюме

У багатьох прикладних задачах виникає потреба будувати опуклі множини, але якщо ці множини отримуються, як розв'язок математичної моделі, то найчастіше використовується поняття опорної функції. Хоча ця функція визначається тільки для опуклих множин, її можна побудувати також і для опуклих оболонок компактних множин. За допомогою цієї функції можна на просторі опуклих компактів вводити поняття диференціювання та інтегрування. Виникає задача побудови опуклої оболонки множини, якщо відомі значення її опорної функції у деяких напрямках. Для цього можуть бути застосовані три алгоритми. Перший використовує властивість опорної функції і будує множину через перети визначених опорною функцією гіперплощин. Другий алгоритм використовує значення функціоналу Мінковського, який пов'язаний з опорною функцією. Третій будує множину по значенням функції деформації, яка також пов'язана зі значеннями опорної функції. У цій статті ставиться задача порівняти швидкість побудови опуклої для цих трьох алгоритмів. Для розрахунків використовувався процесор Intel Core i5-13450HX. Також у цій статті реалізовано числовий метод побудови розв'язку для рівнянь з множиннозначною правою частиною, аналогічний до методу Ейлеру.
Ключові слова: опорна функція, функція деформації, функціонал Мінковського, опуклі множини, диференціальні рівняння із множиннозначною правою частиною, похідна Хукухари, алгоритм Ейлера.

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