UDC 539.3

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DIRECT SOLUTION OF THE DYNAMICAL ELASTICITY PROBLEM FOR A QUARTER SPACE

The wave field of an elastic quarter space is constructed when one face is rigidly fixed and a dynamic normal compressive load is concentrated at the point on another face. The problem was solved by the direct application of the integral Laplace and Fourier transforms to the motion equations and the boundary conditions. This operation leads to the one-dimensional vector inhomogeneous boundary value problem with respect to unknown displacement's transformants. The problem was solved using the matrix differential calculus. A fundamental matrix and a decreasing solution to the corresponding homogenous matrix equation were constructed with a basic residue theorem. A singular integral equation was obtained in the process by satisfying unrealized boundary condition. Weakly convergent part of the equation was summed up. The behavior of the unknown function had been analyzed based on its mechanical sense. The form of unknown function was expressed as a series on Laguerre polynomials. The original displacements' field was found after an application the inverse integral transforms.

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Introduction

An object such as a quarter space can be considered as a model for approbation different approaches of solving the boundary problems of the elasticity theory in static or dynamic statements before considering problems for the infinite and finite plates.

One of the earliest works on the analyses of a three-dimensional wedge problem can be found in works of Uflyand Y.S. [9]. The formulation of the Paphovich-Neuber potential functions was developed for a general wedge by Uflyand Y.S. in [10].

Steady state harmonic vibrations and waves in the elastic bodies considered in the book of Grinchenko V.T. and Meleshko V.V. [3], which provides a broad review of the literature on oscillatory processes.

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In the work [13] Zhang Z., Wang W. and Wong P. presented an explicit matrix algorithm for solving 3D wedge problems under general surface loads: arbitrarily distributed normal and shear loads. The arthors also discussed the effect of wedge angle on internal stresses. A fast and convenient algorithm for the solution of the elastic quarter-space contact problem was presented, which uses discretization to form matrices to realize the overlapping solution process for the elastic quarter-space.

In the work [5] Hanson M. T. and Keer L. M. determined the elastic stress and displacement fields in a quarter-space under arbitrarily applied surface loadings. The problem was formulated in terms of two coupled two-dimensional integral equations. The integral equations contain a logarithmic singularity with an unknown coefficient, which varies along the edge of the quarter-space.

In the paper [2], Babeshko V.A., et. constructed an exact solution in the first quadrant of a plane boundary value problem for the dynamic Lamé elasticity equations, using the coordinate block element method, and expanded the solution in terms of solutions of the boundary value problems for the Helmholtz equation.

Alterman Z.S. and Rotenberg A. in [1] investigated the propagation of the elastic waves for the case of an elastic quarter plane using the finite difference scheme. A point-source emitting a compressional pulse was located along the diagonal inside the quarter plane. Free-surface conditions were assumed on the boundary lines, so that the problem was nonseparable. Complete theoretical seismograms for the horizontal and vertical components of displacement were obtained. The effect of different finite difference formulations for the boundary conditions and the effect of different mesh sizes were studied.

New method of solving the spatial elasticity problems was proposed by Popov G. Ya. in [6]. The method is based on introduction of two new functions which expressed through the derivatives of unknown displacement. The application of this approach leads to the system of two equations and separately solved equation. Using this method an exact solution for elasticity problem for quarter space was obtained in [7; 11] in the static statement by Vaysfeld N.D. and Popov G.Ya.

The dynamical problem for an elastic quarter space was considered in [12] using the method, proposed by Popov G.Ya. In this case the unknown vector of transformants of the displacement consists of two components, that is why implementation of the matrix differential calculus operates with matrices of the 2x2 order.

The aim of this work is to solve the elasticity problem for the quarter space in the dynamic statement directly with the method of the integral transforms and evaluate difficulties and advantages that appear in both approaches. Note that in [12] the problem was solved under assumption that unknown functions are equal to zero, which made the problem basically equivalent to the Lamb problem, and there was no necessity to solve an integral equation.

Main Results

1. Statement of the problem

An elastic quarter space $0 < x,\ z < \infty,\ -\infty < y < \infty$ with μ — Poisson ratio, G — shear modulus is under consideration. Normal dynamic compressive load is acting on a boundary $z = 0$, concentrated at the point with coordinates (a, b) while a boundary $x = 0$ is rigidly fixed. Displacements, $u_x(x, y, z, t) = u(x, y, z, t),\ u_y(x, y, z, t) = v(x, y, z, t),\ u_z(x, y, z, t) = w(x, y, z, t)$, which appear in the medium are in the interest of investigation. The statement leads to the following boundary conditions

$$\begin{gathered} \sigma_z|_{z=0} = \delta(x-a)\delta(y-b)P(t),\ \tau_{zy}|_{z=0} = 0,\ \tau_{zx}|_{z=0} = 0, \\ u|_{x=0} = 0,\ v|_{x=0} = 0,\ w|_{x=0} = 0. \end{gathered} \tag{1}$$

Here $\delta(x)$ is the Dirac delta function. Zero initial conditions are assumed. Unknown displacements satisfy the motion equations, written in a vector form

$$\Delta(u, x, w) + \frac{2}{\kappa - 1}\left(\frac{\partial\Theta}{\partial x}, \frac{\partial\Theta}{\partial y}, \frac{\partial\Theta}{\partial z},\right) = \frac{\rho}{G}\left(\frac{\partial^2 u}{\partial t^2}, \frac{\partial^2 v}{\partial t^2}, \frac{\partial^2 w}{\partial t^2}\right). \tag{2}$$

Here Δ — Laplace operator, $\kappa = 3 - 4\mu$, ρ — density of the elastic medium, $\Theta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$ — volume expansion. The following change of variables is introduced

$$\tilde{x} = x/a,\ \tilde{y} = (y - b)/a,\ \tilde{z} = z/a.$$

In this coordinate system axis $\tilde{y}$ will be a line of symmetry and condition of parity for vertical displacement take place $w(\tilde{x}, -\tilde{y}, \tilde{z}, t) = w(\tilde{x}, \tilde{y}, \tilde{z}, t)$. Normal load (1) in these coordinates take form $\sigma_z|_{z=0} = a^{-2}\delta(x-1)\delta(y)P(t)$. Further symbol “waves” are omitted, implying the change of variables.

2. Reduction the problem to a vector one-dimensional problem.

To reduce the given problem (1)-(2) to the one-dimensional one, the integral transforms are applied, sin-cos- Furrier — with respect to a variable x, Furrier — with respect to a variable y and Laplace — with respect to a variable t

$$\begin{bmatrix} u_{\alpha\beta p}(z) \\ v_{\alpha\beta p}(z) \\ w_{\alpha\beta p}(z) \end{bmatrix} = \int_0^\infty \int_{-\infty}^\infty \int_0^\infty \begin{bmatrix} u(x,y,z,t) \\ v(x,y,z,t) \\ w(x,y,z,t) \end{bmatrix} \begin{pmatrix} \cos\alpha x \\ \sin\alpha x \\ \sin\alpha x \end{pmatrix} e^{i\beta y} e^{-p_* t} dx dy dt \qquad (3)$$

during this operation initial conditions were satisfied. Motion equations (2) and boundary conditions (1) take form

$$\begin{cases} u''_{\alpha\beta p}(z) - \left(\frac{\kappa+1}{\kappa-1}\alpha^2 + \beta^2 + p^2\right) w_{\alpha\beta p}(z) + \frac{2}{\kappa-1}\alpha w'_{\alpha\beta p}(z) - \\ \qquad -\frac{2}{\kappa-1} i\alpha\beta v_{\alpha\beta p}(z) = \frac{\kappa+1}{\kappa-1} u'_{\beta p}(0,z) \\ v''_{\alpha\beta p}(z) - \left(\alpha^2 + \frac{\kappa+1}{\kappa-1}\beta^2 + p^2\right) v_{\alpha\beta p}(z) - \frac{2}{\kappa-1} i\beta w'_{\alpha\beta p}(z) + \\ \qquad +\frac{2}{\kappa-1} i\alpha\beta u_{\alpha\beta p}(z) = 0 \\ w''_{\alpha\beta p}(z) - \frac{\kappa-1}{\kappa+1}\left(\alpha^2 + \beta^2 + p^2\right) w_{\alpha\beta p}(z) - \frac{2}{\kappa+1} i\beta v'_{\alpha\beta p}(z) - \\ \qquad -\frac{2}{\kappa+1} a u'_{\alpha\beta p}(z) = 0 \\ 0 < z < \infty, \end{cases} \qquad (4)$$

Here notation was used $p = p_*/c_2$, p_* — parameter Laplace transform, $c_2 = \sqrt{G/\rho}$ — propagation speed of transverse (shear, secondary or S-) waves.

$$\begin{gathered} \alpha w_{\alpha\beta p}(0) + u'_{\alpha\beta p}(0) = 0, \ -i\beta w_{\alpha\beta p}(0) + v'_{\alpha\beta p}(0) = 0, \\ -\tfrac{3-\kappa}{\kappa+1}\left[\alpha u_{\alpha\beta p}(0) + i\beta v_{\alpha\beta p}(0)\right] + w'_{\alpha\beta p}(0) = \tfrac{\kappa-1}{\kappa+1}\tfrac{P_p}{Ga}\sin\alpha \\ P_p = \int_0^\infty P(t) e^{-p_* t} dt. \end{gathered} \qquad (5)$$

Boundary condition $u(0,y,z,t) = 0$ was not realized. Let's denote unknown function at right part of equations (4)

$$\chi_\beta(z) = u'_{\beta p}(0,z). \qquad (6)$$

In order to rewrite the system (4) in a vector form, the unknown vector of transformants and the vector of right part are introduced

$$\mathbf{y}(z) = \left(u_{\alpha\beta p}(z) \ \ v_{\alpha\beta p}(z) \ \ w_{\alpha\beta p}(z)\right)^T, \mathbf{f}(z) = \left(\frac{\kappa+1}{\kappa-1}\chi_\beta(z) \ \ 0 \ \ 0\right)^T, \qquad (7)$$

and also constant matrices

$$\mathbf{Q}=\begin{pmatrix} 0 & 0 & \frac{1}{\kappa-1}\alpha \\ 0 & 0 & -\frac{1}{\kappa-1}i\beta \\ -\frac{1}{\kappa+1}\alpha & -\frac{1}{\kappa+1}i\beta & 0 \end{pmatrix}$$

$$\mathbf{P}=\begin{pmatrix} -\left(\frac{\kappa+1}{\kappa-1}\alpha^2+\beta^2+p^2\right) & -\frac{2}{\kappa-1}i\alpha\beta & 0 \\ \frac{2}{\kappa-1}i\alpha\beta & -\left(\alpha^2+\frac{\kappa+1}{\kappa-1}\beta^2+p^2\right) & 0 \\ 0 & 0 & -\frac{\kappa-1}{\kappa+1}\left(\alpha^2+\beta^2+p^2\right) \end{pmatrix}$$

So, the system (4) is written using the differential operator of the 2d kind

$$L_2\mathbf{y}(z)=\mathbf{I}\mathbf{y}''(z)+2\mathbf{Q}\mathbf{y}'(z)+\mathbf{P}\mathbf{y}(z)=\mathbf{f}(z),\ 0<z<\infty \tag{8}$$

So, the vector one-dimensional boundary problem was obtained in form (8), (5) with respect to unknown vector of transformants (7).

3. Solving the vector boundary problem.

Solution to the vector problem (8), (5) was constructed in a form [8]

$$\mathbf{y}(z)=\int_0^\infty \mathbf{\Phi}(z-\xi)\mathbf{f}(\xi)d\xi+\mathbf{Y}_-(z)\mathbf{C}, \tag{9}$$

where $\mathbf{\Phi}(z-\xi)$ — fundamental matrix, $\mathbf{C}=(C_1\ \ C_2\ \ C_3)^T$ — constant vector, should be found from the boundary conditions (5), $\mathbf{Y}_-(z)$ — general decreasing, when $z\to\infty$, solution to a matrix equation $L_2\mathbf{Y}(z)=0$.

Characteristic matrix to the equation (8) has a form

$$\mathbf{M}(s)=\begin{pmatrix} s^2-\left(\frac{\kappa+1}{\kappa-1}\alpha^2+\beta^2+p^2\right) & -\frac{2}{\kappa-1}i\alpha\beta & \frac{2}{\kappa-1}\alpha s \\ \frac{2}{\kappa-1}i\alpha\beta & s^2-\left(\alpha^2+\frac{\kappa+1}{\kappa-1}\beta^2+p^2\right) & -\frac{2}{\kappa-1}i\beta s \\ -\frac{2}{\kappa+1}\alpha s & -\frac{2}{\kappa+1}i\beta s & s^2-\frac{\kappa-1}{\kappa+1}(\alpha^2+\beta^2+p^2) \end{pmatrix}.$$

The decreasing solution

$$\mathbf{Y}_-(z)=\frac{1}{2\pi i}\oint_C e^{sz}\mathbf{M}^{-1}(s)ds$$

was constructed on the basis of poles

$$s_1 = -\sqrt{N^2 + \frac{\kappa - 1}{\kappa + 1} p^2}, \quad s_2 = -\sqrt{N^2 + p^2}, \quad N^2 = \alpha^2 + \beta^2,$$

where s_1 is a pole of a first kind, s_2 is a pole of a second kind, of an inverse characteristic matrix

$$\mathbf{M}^{-1}(s) = \frac{\mathbf{M}^A(s)}{\det \mathbf{M}(s)}, \quad \mathbf{M}^A(s) = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}$$

$$A_{11} = \left[s^2 - \left(\frac{\kappa - 1}{\kappa + 1} \alpha^2 + \beta^2 + \frac{\kappa - 1}{\kappa + 1} p^2 \right) \right] [s^2 - (N^2 + p^2)],$$

$$A_{12} = \frac{2}{\kappa + 1} i\alpha\beta [s^2 - (N^2 + p^2)], A_{13} = -\frac{2}{\kappa - 1} \alpha s [s^2 - (N^2 + p^2)],$$

$$A_{21} = -\frac{2}{\kappa + 1} i\alpha\beta [s^2 - (N^2 + p^2)],$$

$$A_{22} = \left[s^2 - \left(\alpha^2 + \frac{\kappa - 1}{\kappa + 1} \beta^2 + \frac{\kappa - 1}{\kappa + 1} p^2 \right) \right] [s^2 - (N^2 + p^2)],$$

$$A_{23} = \frac{2}{\kappa - 1} i\beta s [s^2 - (N^2 + p^2)], \; A_{31} = \frac{2}{\kappa + 1} \alpha s [s^2 - (N^2 + p^2)],$$

$$A_{32} = \frac{2}{\kappa + 1} i\beta s [s^2 - (N^2 + p^2)],$$

$$A_{33} = \left[s^2 - \frac{\kappa + 1}{\kappa - 1} \left(\alpha^2 + \beta^2 + \frac{\kappa - 1}{\kappa + 1} p^2 \right) \right] [s^2 - (N^2 + p^2)],$$

$$\det \mathbf{M} = \left[s^2 - \left(\sqrt{N^2 + p^2} \right)^2 \right]^2 \left[s^2 - \left(\sqrt{N^2 + \frac{\kappa - 1}{\kappa + 1} p^2} \right)^2 \right]$$

and have a following form

$$\mathbf{Y}_-(z) = \mathbf{Y}_-^{(s_1)}(z) + \mathbf{Y}_-^{(s_2)}(z), \tag{10}$$

$$\mathbf{Y}_{-}(z) = \frac{1}{2p^2}e^{-\Delta_1 z}\begin{pmatrix} \dfrac{\alpha^2}{\Delta_1} & \dfrac{i\alpha\beta}{\Delta_1} & \frac{\kappa+1}{\kappa-1}\alpha \\ -\dfrac{i\alpha\beta}{\Delta_1} & \dfrac{\beta^2}{\Delta_1} & -\frac{\kappa+1}{\kappa-1}i\beta \\ -\alpha & -i\beta & -\frac{\kappa+1}{\kappa-1}\Delta_1 \end{pmatrix} +$$

$$+\frac{1}{2p^2}e^{-\Delta_2 z}\begin{pmatrix} -\dfrac{\alpha^2+p^2}{\Delta_2} & -\dfrac{i\alpha\beta}{\Delta_2} & -\frac{\kappa+1}{\kappa-1}\alpha \\ \dfrac{i\alpha\beta}{\Delta_2} & -\dfrac{\beta^2+p^2}{\Delta_2} & \frac{\kappa+1}{\kappa-1}i\beta \\ \alpha & i\beta & \frac{\kappa+1}{\kappa-1}\dfrac{\alpha^2+\beta^2}{\Delta_2} \end{pmatrix},$$

functions Δ_1, Δ_2 have a form

$$\Delta_1 = \sqrt{N^2 + \frac{\kappa-1}{\kappa+1}p^2}, \quad \Delta_2 = \sqrt{N^2+p^2}. \tag{11}$$

To construct a fundamental matrix, the definition can be used. According to it, the solution to the equation $L_2\mathbf{y}(z) = \mathbf{f}(z),\ 0 < z < \infty$, related to (8), can be written in a form

$$\mathbf{y}(z) = \int\limits_0^\infty \mathbf{f}(\xi)\mathbf{\Phi}(z-\xi)d\xi,$$

$\mathbf{\Phi}(z-\xi)$ — fundamental matrix. To solve the equation, continue the right part of it by zero for $z < 0$ and apply the integral Furrier transform

$$\mathbf{y}_s = \int\limits_{-\infty}^\infty e^{isz}\mathbf{y}(z)dz$$

Taking into account, that from a relation $L_2 e^{sz}\mathbf{I} = e^{sz}\mathbf{M}(z)$ follows $L_2 e^{isz}\mathbf{I} = e^{isz}\mathbf{M}(is)$

$$L_2\int\limits_{-\infty}^\infty e^{isz}\mathbf{y}(z)dz = \int\limits_{-\infty}^\infty e^{is\xi}\mathbf{f}(\xi)d\xi,$$

$$\int\limits_{-\infty}^\infty e^{isz}\mathbf{M}(is)\mathbf{y}(z)dz = \mathbf{f}_s,\ \mathbf{M}(is)\mathbf{y}_s = \mathbf{f}_s,\ \mathbf{y}_s = \mathbf{M}^{-1}(is)\mathbf{f}_s.$$

After an application the inverse integral transform to the last relation

$$\mathbf{y}(z) = \frac{1}{2\pi}\int\limits_{-\infty}^\infty \mathbf{M}^{-1}(is)\mathbf{f}_s e^{-isz}ds = \int\limits_0^\infty \mathbf{f}(\xi)\left(\frac{i}{2\pi}\int\limits_{-\infty}^\infty \mathbf{M}^{-1}(is)e^{-is(z-\xi)}ds\right)d\xi$$

and comparing it with the definition, resulting formula was obtained

$$\mathbf{\Phi}(z-\xi)=\frac{i}{2\pi}\int\limits_{-\infty}^{\infty}\mathbf{M}^{-1}(is)e^{-is(z-\xi)}ds.$$

Characteristic matrix has a form

$$\mathbf{M}(is)=\begin{pmatrix} -s^2-\left(\frac{\kappa+1}{\kappa-1}\alpha^2+\beta^2+p^2\right) & -\frac{2}{\kappa-1}i\alpha\beta \\ \frac{2}{\kappa-1}i\alpha\beta & -s^2-\left(\alpha^2+\frac{\kappa+1}{\kappa-1}\beta^2+p^2\right) \\ \frac{2}{\kappa+1}i\alpha s & -\frac{2}{\kappa+1}i\beta s \end{pmatrix.$$

$$\begin{matrix} -\frac{2}{\kappa-1}i\alpha s \\ -\frac{2}{\kappa-1}\beta s \\ -s^2-\frac{\kappa-1}{\kappa+1}(\alpha^2+\beta^2+p^2)\end{pmatrix},$$

$$\mathbf{M}^{-1}(is)=\frac{\mathbf{M}^A(is)}{\det\mathbf{M}(is)},\mathbf{M}^A(is)=\begin{pmatrix}A_{11}&A_{12}&A_{13}\\A_{21}&A_{22}&A_{23}\\A_{31}&A_{32}&A_{33}\end{pmatrix},$$

$$A_{11}=\left[s^2+\left(\frac{\kappa-1}{\kappa+1}\alpha^2+\beta^2+\frac{\kappa-1}{\kappa+1}p^2\right)\right][s^2+N^2+p^2]$$

$$A_{12}=-\frac{2}{\kappa+1}i\alpha\beta[s^2+N^2+p^2],\ A_{13}=-\frac{2}{\kappa-1}i\alpha s[s^2+N^2+p^2],$$

$$A_{21}=-\frac{2}{\kappa+1}i\alpha\beta[s^2+N^2+p^2],$$

$$A_{22}=\left[s^2+\left(\alpha^2+\frac{\kappa-1}{\kappa+1}\beta^2+\frac{\kappa-1}{\kappa+1}p^2\right)\right][s^2+N^2+p^2],$$

$$A_{23}=\frac{2}{\kappa-1}i\beta s[s^2-(N^2+p^2)],A_{31}=\frac{2}{\kappa+1}i\alpha s[s^2+N^2+p^2],$$

$$A_{32}=-\frac{2}{\kappa+1}\beta s[s^2+N^2+p^2],$$

$$A_{33}=\left[s^2+\frac{\kappa+1}{\kappa-1}\left(\alpha^2+\beta^2+\frac{\kappa-1}{\kappa+1}p^2\right)\right][s^2+N^2+p^2],$$

$$\det\mathbf{M}(is)=\left[s-i\sqrt{N^2+p^2}\right]^2\left[s+i\sqrt{N^2+p^2}\right]^2\cdot$$
$$\cdot\left[s-i\sqrt{N^2}+\tfrac{\kappa-1}{\kappa+1}p^2\right]\left[s+i\sqrt{N^2+\tfrac{\kappa-1}{\kappa+1}p^2}\right]$$

$$s_1=-i\sqrt{N^2+\frac{\kappa-1}{\kappa+1}p^2},\ s_2=-i\sqrt{N^2+p^2},$$

$$s_3 = i\sqrt{N^2 + \frac{\kappa-1}{\kappa+1}p^2},\ s_4 = i\sqrt{N^2+p^2}$$

Using Jordan lemma, closing contours in the upper and lower half-planes, the main theorem on residues, taking into account the kinds of poles, resulting formula take a form

$$\frac{i}{2\pi}\int\limits_{-\infty}^{\infty}\mathbf{M}^{-1}(is)e^{-is(z-\xi)}ds = \left[\begin{array}{l} i[\mathrm{Res}(s_3)+\mathrm{Res}(s_4)],\ z-\xi<0 \\ -i[\mathrm{Res}(s_1)+\mathrm{Res}(s_2)], z-\xi>0 \end{array}\right.$$

Fundamental matrix was constricted

$$\mathbf{\Phi}(z-\xi) = \tfrac{1}{2p^2}e^{-\Delta_2|z-\xi|}\begin{pmatrix} -\dfrac{\alpha^2+p^2}{\Delta_2} & \dfrac{i\alpha\beta}{\Delta_2} & \mathrm{sgn}(z-\xi)\frac{\kappa+1}{\kappa-1}\alpha \\ \dfrac{i\alpha\beta}{\Delta_2} & \dfrac{\beta^2+p^2}{\Delta_2} & -\mathrm{sgn}(z-\xi)\frac{\kappa+1}{\kappa-1}i\beta \\ \mathrm{sgn}(z-\xi)\alpha & -\mathrm{sgn}(z-\xi)i\beta & -\frac{\kappa+1}{\kappa-1}\dfrac{\alpha^2+\beta^2}{\Delta_2} \end{pmatrix} +$$

$$+\frac{1}{2p^2}e^{-\Delta_1|z-\xi|}\begin{pmatrix} \dfrac{\alpha^2}{\Delta_1} & -\dfrac{i\alpha\beta}{\Delta_1} & -\mathrm{sgn}(z-\xi)\frac{\kappa+1}{\kappa-1}\alpha \\ -\dfrac{i\alpha\beta}{\Delta_1} & -\dfrac{\beta^2}{\Delta_1} & \mathrm{sgn}(z-\xi)\frac{\kappa+1}{\kappa-1}i\beta \\ -\mathrm{sgn}(z-\xi)\alpha & \mathrm{sgn}(z-\xi)i\beta & \frac{\kappa+1}{\kappa-1}\Delta_1 \end{pmatrix}$$

The solution (9) can be rewritten in a matrix form

$$\begin{aligned} \begin{pmatrix} u_{\alpha\beta p}(z) \\ v_{\alpha\beta p}(z) \\ w_{\alpha\beta p}(z) \end{pmatrix} &= \frac{1}{2p^2}\int\limits_0^{\infty}\begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} \\ \phi_{21} & \phi_{22} & \phi_{23} \\ \phi_{31} & \phi_{32} & \phi_{33} \end{pmatrix}\begin{pmatrix} \frac{\kappa+1}{\kappa-1}\chi_\beta(\xi) \\ 0 \\ 0 \end{pmatrix}d\xi + \\ &+\frac{1}{2p^2}\begin{pmatrix} y_-^{11} & y_-^{12} & y_-^{13} \\ y_-^{21} & y_-^{22} & y_-^{23} \\ y_-^{31} & y_-^{32} & y_-^{33} \end{pmatrix}\begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix} \end{aligned} \tag{12}$$

So, transformants of displacements can be written as

$$\begin{aligned} u_{\alpha\beta p}(z) &= \tfrac{\kappa+1}{\kappa-1}\frac{1}{2p^2}\int\limits_0^{\infty}\phi_{11}\chi_\beta(\xi)d\xi + \frac{1}{2p^2}(y_-^{11}C_1 + y_-^{12}C_2 + y_-^{13}C_3) \\ v_{\alpha\beta p}(z) &= \tfrac{\kappa+1}{\kappa-1}\frac{1}{2p^2}\int\limits_0^{\infty}\phi_{21}\chi_\beta(\xi)d\xi + \frac{1}{2p^2}(y_-^{21}C_1 + y_-^{22}C_2 + y_-^{23}C_3) \\ w_{\alpha\beta p}(z) &= \tfrac{\kappa+1}{\kappa-1}\frac{1}{2p^2}\int\limits_0^{\infty}\phi_{31}\chi_\beta(\xi)d\xi + \frac{1}{2p^2}(y_-^{31}C_1 + y_-^{32}C_2 + y_-^{33}C_3) \end{aligned} \tag{13}$$

where

$$\phi_{11} = -\frac{\alpha^2 + p^2}{\Delta_2} e^{-\Delta_2|z-\xi|} + e^{-\Delta_1|z-\xi|}\frac{\alpha^2}{\Delta_1},$$
$$\phi_{21} = i\alpha\beta\left[\frac{1}{\Delta_2}e^{-\Delta_2|z-\xi|} - \frac{1}{\Delta_1}e^{-\Delta_1|z-\xi|}\right],$$
$$\phi_{31} = \alpha\,\mathrm{sgn}(z-\xi)[e^{-\Delta_2|z-\xi|} - e^{-\Delta_1|z-\xi|}]$$

Components y_-^{ij} of the decreasing solution are given in (10). To find unknown constants in the solution (13), a system of equations was obtained after satisfying the boundary conditions (5).

$$2\tfrac{\kappa-1}{\kappa+1}\alpha\left[\frac{N^2+\frac{1}{2}p^2}{\Delta_1} - \Delta_2\right]C_1 + 2\tfrac{\kappa-1}{\kappa+1}i\beta\left[\frac{N^2+\frac{1}{2}p^2}{\Delta_1} - \Delta_2\right]C_2 + p^2C_3 =$$
$$= \tfrac{\kappa-1}{\kappa+1}\tfrac{2p^2}{Ga}\sin\alpha P_p + B_1$$
$$p^2C_1 - 2\tfrac{\kappa+1}{\kappa-1}\alpha\left[\Delta_1 - \frac{N^2+\frac{1}{2}p^2}{\Delta_2}\right]C_3 = B_2$$
$$p^2C_2 + 2\tfrac{\kappa+1}{\kappa-1}i\beta\left[\Delta_1 - \frac{N^2+\frac{1}{2}p^2}{\Delta_2}\right]C_3 = B_3$$

Right part of equations has a form

$$B_1 = \alpha\int_0^\infty \chi_\beta(\xi)\left[2\Delta_2 e^{-\Delta_2\xi} - \frac{2N^2+p^2}{\Delta_1}e^{-\Delta_1\xi}\right]d\xi,$$
$$B_2 = \tfrac{\kappa+1}{\kappa-1}\int_0^\infty \chi_\beta(\xi)[(2\alpha^2+p^2)e^{-\Delta_2\xi} - 2\alpha^2 e^{-\Delta_1\xi}]d\xi,$$
$$B_3 = 2\tfrac{\kappa+1}{\kappa-1}i\alpha\beta\int_0^\infty \chi_\beta(\xi)[-e^{-\Delta_2\xi} + e^{-\Delta_1\xi}]d\xi.$$

Determinant of the system is

$$\det = \frac{p^2}{\Delta_1\Delta_2}[\Delta_1\Delta_2 - N^2]\Delta,$$
$$\Delta = 4N^4 + 4N^2p^2 + p^4 - 4N^2\Delta_1\Delta_2. \tag{14}$$

After solving the system constants were found

$$
\begin{gathered}
C_1=\tfrac{2p^2}{Ga}\alpha\sin\alpha P_p\frac{\Delta_1[\Delta_1\Delta_2-(N^2+\frac{1}{2}p^2)]}{[\Delta_1\Delta_2-N^2]\Delta}+C_1^B,\\
C_2=-\tfrac{2p^2}{Ga}i\beta\sin\alpha P_p\frac{\Delta_1[\Delta_1\Delta_2-(N^2+\frac{1}{2}p^2)]}{[\Delta_1\Delta_2-N^2]\Delta}+C_2^B,\\
C_3=\tfrac{\kappa-1}{\kappa+1}\tfrac{p^4}{Ga}\sin\alpha P_p\frac{\Delta_1\Delta_2}{[\Delta_1\Delta_2-N^2]\Delta}+C_3^B,\\
C_1^B=\tfrac{\kappa+1}{\kappa-1}\frac{1}{[\Delta_1\Delta_2-N^2]\Delta}\int\limits_0^\infty\chi_\beta(\xi)[\alpha^2(\Delta-4p^2\Delta_1\Delta_2)e^{-\Delta_2\xi}+\\
+(p^4\Delta_1\Delta_2+2\alpha^2p^2\Delta_1\Delta_2)e^{-\Delta_1\xi}]d\xi,\\
C_2^B=\tfrac{\kappa+1}{\kappa-1}\frac{i\alpha\beta}{[\Delta_1\Delta_2-N^2]\Delta}\int\limits_0^\infty\chi_\beta(\xi)[(4p^2\Delta_1\Delta_2-\Delta)e^{-\Delta_2\xi}+\\
+(4p^4\Delta_1\Delta_2+8N^2\Delta_1\Delta_2-\Delta)e^{-\Delta_1\xi}]d\xi,\\
C_3^B=\frac{\alpha\Delta_2}{[\Delta_1\Delta_2-N^2]\Delta}\int\limits_0^\infty\chi_\beta(\xi)[(4N^4-4N^2\Delta_1\Delta_2-p^4)e^{-\Delta_2\xi}+\\
+(4p^4\Delta_1\Delta_2-\Delta)e^{-\Delta_1\xi}]d\xi.
\end{gathered}
\tag{15}
$$

Transformants of the displacements can be written in a following form

$$
(u_{\alpha\beta p}(z)\ \ v_{\alpha\beta p}(z)\ \ w_{\alpha\beta p}(z))^T=(u^0_{\alpha\beta p}(z)\ \ v^0_{\alpha\beta p}(z)\ \ w^0_{\alpha\beta p}(z))^T+
$$

$$
+(u^1_{\alpha\beta p}(z)\ \ v^1_{\alpha\beta p}(z)\ \ w^1_{\alpha\beta p}(z))^T
$$

Where additions $u^0_{\alpha\beta p}(z), v^0_{\alpha\beta p}(z), w^0_{\alpha\beta p}(z)$ correspond to the solution, which include intensity of the acting load P_p, and additions with an upper index "1" include an integral with the unknown function $\chi_\beta(\xi)$ in (6). Transformants with an index "0" have a form

$$
\begin{gathered}
u^0_{\alpha\beta p}(z)=\alpha\frac{\sin\alpha}{Ga}\frac{P_p}{\Delta}[2\Delta_1\Delta_2e^{-\Delta_2z}-(2N^2+p^2)e^{-\Delta_1z}],\\
v^0_{\alpha\beta p}(z)=i\beta\frac{\sin\alpha}{Ga}\frac{P_p}{\Delta}[2\Delta_1\Delta_2e^{-\Delta_2z}-(2N^2+p^2)e^{-\Delta_1z}]\\
w^0_{\alpha\beta p}(z)=\frac{\sin\alpha}{Ga}P_p\frac{\Delta_1}{\Delta}[2N^2e^{-\Delta_2z}-(2N^2+p^2)e^{-\Delta_1z}].
\end{gathered}
\tag{16}
$$

It can be easily seen that transformants coincide with ones, obtained with the method of Popov G.Ya. in [12]. Difference is in sign, because a compressive load is here in the statement of the problem in contrast to a stretching load in that investigation. Using the relation (13), so as components of decreasing solution (10) and constants (15), transformant of interest for displacement was constructed

$$
\begin{aligned}
w_{\alpha\beta p}(z) = \tfrac{\kappa+1}{\kappa-1}\frac{\alpha}{2p^2}\int\limits_0^\infty [e^{-\Delta_2|z-\xi|}\operatorname{sgn}(z-\xi)-\\
-e^{-\Delta_1|z-\xi|}\operatorname{sgn}(z-\xi)]\chi_\beta(\xi)d\xi+\\
+\tfrac{\kappa+1}{\kappa-1}\frac{\alpha}{2p^2\Delta}\int\limits_0^\infty F_1(N,z,\xi,p)\chi_\beta(\xi)d\xi + w^0_{\alpha\beta p(z)}.
\end{aligned}
\tag{17}
$$

To find the initial vertical displacement the inverse integral transform should be applied

$$
w(x,y,z,t) = \frac{1}{\pi^2}\frac{1}{2\pi i}\int\limits_l \int\limits_{-\infty}^{\infty}\int\limits_0^{\infty} w_{\alpha\beta p}(z)e^{p_* t}e^{i\beta y}\sin\alpha x dp_* d\beta d\alpha,
$$

$l = (\lambda - i\infty, \lambda + i\infty)$ Algorithm of conducting a component $w_0(x,y,z,t)$ was considered in [12], and related to the solution of the Lamb problem.

4. Reduction to the singular integral equation.

An integral equation was constructed, based on the fact, that the boundary condition $u(0,y,z,t) = 0$ in (1) has not been realized yet. Considering the solution $u_{\alpha\beta p}(z)$ in (13), rewrite it in the form

$$
\begin{aligned}
u_{\alpha\beta p}(z) = \tfrac{\kappa+1}{\kappa-1}\frac{1}{2p^2}\int\limits_0^\infty \left[\frac{\alpha^2}{\Delta_1}e^{-\Delta_1|z-\xi|} - \frac{\alpha^2}{\Delta_2}e^{-\Delta_2|z-\xi|}\right]\chi_\beta(\xi)d\xi-\\
-\tfrac{\kappa+1}{\kappa-1}\frac{1}{2p^2}\int\limits_0^\infty \frac{p^2}{\Delta_2}e^{-\Delta_2|z-\xi|}\chi_\beta(\xi)d\xi + \frac{1}{2p^2}(y_-^{11}C_1 + y_-^{12}C_2 + y_-^{13}C_3)
\end{aligned}
$$

or after substitution values for constants (15) and elements of decreasing solution (10)

$$
\begin{aligned}
u_{\alpha\beta p}(z) = \tfrac{\kappa+1}{\kappa-1}\frac{1}{2p^2}\int\limits_0^\infty \left[\frac{\alpha^2}{\Delta_1}e^{-\Delta_1|z-\xi|} - \frac{\alpha^2}{\Delta_2}e^{-\Delta_2|z-\xi|}\right]\chi_\beta(\xi)d\xi+\\
+\tfrac{\kappa+1}{\kappa-1}\frac{1}{2p^2}\int\limits_0^\infty F^1(z,\xi)\chi_\beta(\xi)d\xi + \tfrac{\kappa+1}{\kappa-1}\frac{1}{2p^2}\frac{\alpha^2}{\Delta}\int\limits_0^\infty F^2(z,\xi)\chi_\beta(\xi)d\xi + u^0_{\alpha\beta p}(z),
\end{aligned}
$$

where

$$F^1(z,\xi) = -\frac{p^2}{\Delta_2}e^{-\Delta_2|z-\xi|} - \frac{p^2}{\Delta_2}e^{-\Delta_2 z}e^{-\Delta_2\xi} +$$
$$+N^2\left(\frac{1}{\Delta_1}e^{-\Delta_1 z} - \frac{1}{\Delta_2}e^{-\Delta_2 z}\right) - \Delta_2(e^{-\Delta_1 z} - e^{-\Delta_2 z})e^{-\Delta_2\xi},$$
$$F^2(z,\xi) = \frac{1}{[\Delta_1\Delta_2 - N^2]}(p^2[\Delta_2 e^{-\Delta_1 z} - \Delta_1 e^{-\Delta_2 z}]\cdot$$
$$\cdot[4N^2 e^{-\Delta_1\xi} + (4p^2 + 8N^2)e^{-\Delta_2\xi}] +$$
$$+\Delta_2[e^{-\Delta_1 z} - e^{-\Delta_2 z}][(4N^2 - 4N^2\Delta_1\Delta_2 - p^4)e^{-\Delta_1\xi} + 4p^2\Delta_1\Delta_2 e^{-\Delta_2\xi}])$$

After an application the inverse cosine transform Furrier with respect to the variable α to the solution $u_{\alpha\beta p}(z)$ and changing the order of integration

$$u^1_{\beta p}(x,z) \sim \frac{\kappa+1}{\kappa-1}\frac{1}{2p^2}\frac{2}{\pi}\int_0^\infty \chi_\beta(\xi)\int_0^\infty \left[e^{-\Delta_1|z-\xi|}\frac{\alpha^2}{\Delta_1} - e^{-\Delta_2|z-\xi|}\frac{\alpha^2}{\Delta_2}\right]\cos\alpha x d\alpha d\xi$$

consider the inner integral with respect to the variable α, which can be calculated using the table integral № 3.961(2) in [3]

$$\int_0^\infty \frac{e^{-\sqrt{\alpha^2+\beta^2}|z-\xi|}}{\alpha^2+\beta^2}\cos\alpha x d\alpha = K_0(|\beta|\sqrt{x^2+(z-\xi)^2}),$$

$K_0(z)$ — Macdonald function, taking into account that $\alpha^2\cos\alpha x = -(\cos\alpha x)''_x$ and a form of the functions Δ_1, Δ_2, defined in (11)

$$u^1_{\beta p}(x,z) \sim \tfrac{\kappa+1}{\kappa-1}\frac{1}{\pi p^2}\left(-\frac{\partial^2}{\partial x^2}\right)\int_0^\infty \chi_\beta(\xi)[K_0(\Upsilon_1\sqrt{x^2+(z-\xi)^2}) -$$
$$-K_0(\Upsilon_2\sqrt{x^2+(z-\xi)^2})]d\xi, \tag{18}$$
$$\Upsilon_1 = \sqrt{\beta^2 + \tfrac{\kappa-1}{\kappa+1}p^2}, \quad \Upsilon_2 = \sqrt{\beta^2+p^2}$$

Direct verification can approve the correctness of the relation

$$\frac{\partial^2}{\partial x^2}K_0(\sqrt{\beta^2+p^2}\sqrt{x^2+(z-\xi)^2}) + \frac{\partial^2}{\partial z^2}K_0(\sqrt{\beta^2+p^2}\sqrt{x^2+(z-\xi)^2}) -$$
$$-(\beta^2+p^2)K_0(\sqrt{\beta^2+p^2}\sqrt{x^2+(z-\xi)^2}) = 0$$

After an application this relation to (18), it was rewritten in a form

$$u^1_{\beta p}(x,z) \sim \tfrac{\kappa+1}{\kappa-1}\frac{1}{\pi p^2}\left(\frac{\partial^2}{\partial z^2} - \Upsilon_1^2\right)\int_0^\infty \chi_\beta(\xi)K_0(\Upsilon_1\sqrt{x^2+(z-\xi)^2}d\xi -$$
$$-\tfrac{\kappa+1}{\kappa-1}\frac{1}{\pi p^2}\left(\frac{\partial^2}{\partial z^2} - \Upsilon_2^2\right)\int_0^\infty \chi_\beta(\xi)K_0(\Upsilon_2\sqrt{x^2+(z-\xi)^2})d\xi.$$

Here the value $x = 0$ can be fixed

$$u^1_{\beta p}(0,z) \sim \tfrac{\kappa+1}{\kappa-1}\frac{1}{\pi p^2}\left(\frac{\partial^2}{\partial z^2} - \Upsilon_1^2\right)\int\limits_0^\infty \chi_\beta(\xi)K_0(\Upsilon_1|z-\xi|)d\xi -$$
$$-\tfrac{\kappa+1}{\kappa-1}\frac{1}{\pi p^2}\left(\frac{\partial^2}{\partial z^2} - \Upsilon_2^2\right)\int\limits_0^\infty \chi_\beta(\xi)K_0(\Upsilon_2|z-\xi|)d\xi.$$

So, the integral equation for the unknown function $\chi_\beta(z) = u'_{\beta p}(0,z)$ has been obtained

$$\frac{1}{\pi}\left(\frac{\partial^2}{\partial z^2} - \Upsilon_1^2\right)\int\limits_0^\infty \chi_\beta(\xi)K_0(\Upsilon_1|z-\xi|)d\xi -$$
$$-\frac{1}{\pi}\left(\frac{\partial^2}{\partial z^2} - \Upsilon_2^2\right)\int\limits_0^\infty \chi_\beta(\xi)K_0(\Upsilon_2|z-\xi|)d\xi +$$
$$+\frac{2}{\pi}\int\limits_0^\infty \chi_\beta(\xi)\int\limits_0^\infty\left(-\frac{p^2}{\Delta_2}e^{-\Delta_2|z-\xi|} + \right. \tag{19}$$
$$\left. + \alpha^2 F^1(z,\xi) + \frac{\alpha^2}{\Delta}F^2(z,\xi) + \frac{1}{\Delta}F^3(z,\xi)\right)d\alpha d\xi =$$
$$= \frac{\kappa-1}{\kappa+1}\frac{2p^2P_p}{\pi G a}\int\limits_0^\infty \frac{\alpha\sin\alpha}{\Delta}[(2N^2+p^2)e^{-\Delta_1 z} - 2\Delta_1\Delta_2 e^{-\Delta_2 z}]d\alpha$$

Here Δ is defined in (14), $\Delta_i, i = 1,2$ — in (11), $\Upsilon_i, i = 1,2$ — in (18),

$$[\Delta_1\Delta_2 - N^2]F^1(z,\xi) = -\frac{p^2}{\Delta_2}e^{-\Delta_2 z}e^{-\Delta_1\xi} + \beta^2\left[\frac{1}{\Delta_1}e^{-\Delta_1 z} - \frac{1}{\Delta_2}e^{-\Delta_2 z}\right]e^{-\Delta_2\xi} -$$
$$-N^2\left[\frac{1}{\Delta_1}e^{-\Delta_1 z} - \frac{1}{\Delta_2}e^{-\Delta_2 z}\right]e^{-\Delta_1\xi} - \Delta_2[e^{-\Delta_1 z} - e^{-\Delta_2 z}]e^{-\Delta_2\xi},$$

$$[\Delta_1\Delta_2 - N^2]F^2(z,\xi) = f^1(N)e^{-\Delta_2 z}e^{-\Delta_1\xi} + f^2(N)e^{-\Delta_2 z}e^{-\Delta_2\xi} +$$
$$+ f^3(N)e^{-\Delta_1 z}e^{-\Delta_1\xi} + f^4(N)e^{-\Delta_1 z}e^{-\Delta_2\xi},$$

$$f^1(N) = 4\Delta_1 p^4 + 4N^4p^2\Delta_1 - \Delta_2(4N^2 - 4N^2\Delta_1\Delta_2 - p^4),$$

$$f^2(N) = -p^4\Delta_1\Delta_2 - 4N^2(N^2 + \frac{1}{2}p^2 - \Delta_1\Delta_2) - 4p^2\Delta_1\Delta_2,$$

$$f^3(N) = -4N^4p^2\Delta_2 + \Delta_2(4N^2 - 4N2\Delta_1\Delta_2 - p^4),$$

$$f^4(N) = 4N^2p^2\Delta_1\Delta_2 + p^4\Delta_1\Delta_2 + 4N^2(N^2 + \frac{1}{2}p^2 - \Delta_1\Delta_2) - 4p^2\Delta_1\Delta_2,$$

$$\Delta_2[\Delta_1\Delta_2 - N^2]F^3(z,\xi) = p^2e^{-\Delta_2 z}e^{-\Delta_2\xi}[p^4\Delta_1\Delta_2 - 4\beta^2(N^2 + \frac{1}{2}p^2 - \Delta_1\Delta_2)^2].$$

Analyzing the unknown function in the integral equation (19), it can be seen that

$$\chi_\beta(z) = u'_{\beta p}(0,z) \sim \frac{\partial}{\partial x}u(0,y,z,t) = \left(G\frac{\kappa+1}{\kappa-1}\right)^{-1}\sigma_x(0,y,z)$$

So, the behavior of the function $\chi_\beta(z) = u'_{\beta p}(0,z)$ is the same as a behavior of a normal stress $\sigma_x(0,y,z)$ while $z \to 0$. Normal stress tends to infinity according to a power low with an exponent equal to $-\gamma(\gamma < 1)$, which depends on a Poisson ratio μ of the medium.

The unknown function is represented as series

$$\chi_\beta(\xi) = \sum_{n=1}^{\infty} x_n(\beta)e^{-\xi}\xi^{-\gamma}L_n^{(-\gamma)}(2\xi)$$

where $L_n^{(-\gamma)}(2\xi)$ — Laguerre polynomials, $x_n(\beta)$ unknowns. This correspondence should be substituted into the equation (19) with the following application the method of orthogonal polynomials. After that, the unknown function $\chi_\beta(z)$ should be substituted into the expression for the vertical displacement (17).

Conclusion

The dynamic problem for the elastic quarter space was solved by the direct application of the integral transforms to the motion equations and the boundary conditions. This operation reduces the initial problem to the one-dimensional vector boundary problem, which was solved with the help of a matrix differential calculus. In this case all needed matrices, such as decreasing solution and fundamental matrix turned to be 3×3, so as a system of equations for finding constants in the solution, which significantly complicates calculations, if compare with application of the method of Popov G.Ya., where all needed matrices were 2×2 order. In the process an integral singular equation was derived by satisfying a remaining boundary condition. Components of displacements which include the intensity of the acting load coincide with ones, obtained earlier by the method of Popov G.Ya. The future investigation will be deducted to the analysis of the steady-state oscillations and displacements which appear in the medium under acting distributed load.

Фесенко Г. О.

Безпосереднє розв'язання динамічної задачі для пружного чверть простору

Резюме

Побудовано поле переміщень у пружному чверть простору, коли одна границя жорстко закріплена, а на іншій діє динамічна нормальна стискальна сила, зосереджена у точці. Задачу було розв'язано із застосуванням методу інтегральних перетворень Лапласа та Фур'є безпосередньо до рівнянь руху та граничних умов. Це призводить до одновимірної векторної неоднорідної крайової задачі відносно невідомих трансформант переміщень. Цю задачу розв'язано з допомогою матричного диференціального числення. Фундаментальна матриця та спадаючий розв'язок відповідного матричного рівняння були побудовані з допомогою основної теореми про лишки. Сингулярне інтегральне рівняння отримано у процесі реалізації граничної умови. Слабко збіжна частина рівняння була підсумована із виділенням сингулярного ядра. Поведінку невідомої у інтегральному рівнянні функції було проаналізовано на основі її механічного змісту. Невідому функцію подано у вигляді ряду по поліномам Лагерра. Оригінал вертикального переміщення було отримано після застосування обернених інтегральних перетворень.

Ключові слова: пружний чверть простір, інтегральні перетворення, динамічне навантаження матричне диференціальне числення, сингулярне інтегральне рівняння.

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